The big ABC

For four years, mathematicians have been discussing the alleged proof of an enigmatic conjecture by Shin'ichi Mochizuki from Japan. Still, hardly anyone has understood the work - and perhaps this will never change

By Marlene Weiss

In a children's story written by the Swiss author Peter Bichsel, a lonely man decides to invent his own language. He calls the table "carpet", the chair "alarm clock", the bed "picture". At first he is enthusiastic about his idea and always thinks of new words, his sentences sound original and funny. But after a while, he begins to forget the old words.

   This story comes to Jakob Stix' mind when he thinks of his japanese colleague Shin'ichi Mochizuki. For more than ten years, Mochizuki had worked largely by himself on a proof of the so-called ABC conjecture, one of the most important unresolved problems in mathematics. In the summer of 2012 he was done, and published his proof. But the mathematical realm that he has built up over all these years is so complicated, the language is so strange that hardly anybody except Mochizuki himself can find their way within it. And whether the proof is solid has still not been clarified. "It annoys me very much, I would like to understand it," says Stix, a young number theory specialist at the University of Frankfurt. After all, he's a respected expert himself in the field. "If the result is correct and the proof is done, then this is something big."

   For that reason, it's not as if people hadn't made an effort. An international conference on Mochizuki's work was organized in Oxford in 2015, and a second one took place at the end of July at Mochizuki's institute in Kyoto. Many renowned mathematicians have tried to comprehend the line of thought. That did not lead to much: Probably, it will take years before the proof has been worked through, participants said afterwards. If that ever happens.

Thus, the puzzling ABC-conjecture remains in limbo: not proven, not unproven. And that is even though, or maybe because, it is actually quite simple, by mathematics standards. It's only about basic calculation methods, multiplication and addition. The conjecture, dating from the eighties, concerns the sum called C of two numbers A and B, as well as their decomposition into prime factors - that is, numbers which are no longer divisible. The conjecture is then, in very simple terms: If A and B each contain one prime factor many times, then this usually is not the case for their sum C.

If, for example, one adds $3 \times 3 \times 3 \times 3 \times 5$ to $2 \times 2 \times 2 \times 2 \times 7$, that yields the bulky number 517, which can then only be decomposed as 11×47. So the ABC conjecture says something about what happens to numbers when one adds them - pretty much the simplest thing one can do in mathematics; elementary school, first grade. That's why the conjecture is so fascinating, and so important: if it was proven, many other proofs would simply follow, it would be a deep insight into the secrets of numbers.

   However, this only holds if the proof is valid. But is Mochizuki's work correct? The proof is spread over four scientific articles, which together fill about 500 pages. They, in turn, rely on various earlier works by Mochizuki. More than a thousand pages of dense mathematics, in a notation that hardly anyone understands: Have fun with that. One can not expect every mathematician to work through all of this. "A proof always has a social aspect", says Jakob Stix. "It takes a group of experts, who agree that it is true. And it has to be experts whom one can trust."

So far, this concept has worked well most of the time. In the case of Fermat's last theorem for instance, proven by Andrew Wiles in 1994, another mammoth task. And faulty, in the first version: Wiles had submitted his work for publication in the Annals of Mathematics, one among the two most important mathematics journals in the world. Its editors had Wiles' work thoroughly examined, section by section, by several experts. One of them noticed a treacherous mistake that destroyed the whole proof. But Wiles was able to close the gap, his proof is still considered a milestone.

Whether something similar will occur with Mochizuki's work, however, is questionable. Apparently, he has submitted his proof to one of the japanese journals in which his earlier work has been published. There may well be reasons for that. However, a publication in those journals would not have the same weight as it would have in one of the international flagship journals.

Mochizuki himself does not talk to journalists, and he has not responded to an interview request by Sueddeutsche Zeitung. On his website, where he presents himself as an "Inter-Universal Geometer", Mochizuki dryly informs potential job candidates that the official language at the Institute is Japanese. This may seem to some as an unpleasant obstacle, he concedes, but firstly there is no shortage of English-spoken institutes throughout the world. Moreover, he writes, the Japanese-language based mathematical culture here is "a precious cultural asset, both for Japan and for the world". If the goal should be to keep that world at bay, these words might be quite well chosen.

Meanwhile, the mathematic community is almost hopelessly divided. On one side, there's Mochizuki with his followers; a few claim to have understood the proof. Others are at least convinced of the geniality of the work and continue to dig through the notation. One of them is Taylor Dupuy, a young mathematician at the University of Vermont. With his colleagues, he is organizing another conference on Mochizuki's work in September, his enthusiasm is unbroken. "There are so many new ideas, so much one can do with it," he says. "And it's a lot of work, but we're progressing."

The first two parts of Mochizuki's work are more or less understood, he says, the fourth is, too. Just with the third one, the main part, they're still at the beginning. The meeting in Kyoto was very useful, according to Dupuy. Mochizuki's lectures may be world famous for their unique incomprehensibility. "But he answered questions very well," says Dupuy. Sure, the language is complicated, he says. But some of it could be necessary, if one wants to say something completely new.

However, many experts see that differently. They cannot see why they should invest years of their lives and their careers into this tangle of mathematical notation. Jakob Stix, for example, does not believe that Mochizuki's personal language of mathematics is really necessary.

"Much is said with new, flowery words, above everything lies a layer of obfuscation", he says. "That's what makes it so difficult."

Actually, he says, one should set up a team to investigate the proof. A couple of young, motivated people and a few experts, all working on nothing else for years. Why not? "Mathematical research is so cheap, for other things, we spend billions," says Stix. But as it is, it is a risk to become engrossed in Mochizuki's work, at a venture and all by oneself. Especially if one does not have a permanent job yet. At best, the laurels go to Mochizuki; at worst, everything was in vain.

Gerd Faltings too sounds tired when he's asked about Mochizuki's work. His opinion carries weight within the community, as he is not only a

director at the Max Planck Institute for Mathematics in Bonn and the only German to have won the prestigious Fields Medal, but also Mochizuki's doctoral advisor. In principle, Faltings would not put it past his former student to prove the ABC conjecture. "But if he doesn't put more effort into explaining it, then there's no point, then it will all stay suspended in the air," he says. He didn't even attend the workshop in Kyoto: the first workshop at Oxford had been too disappointing, he says, he hadn't become any wiser there.

Table is called "carpet", chair is called "alarm clock", bed is called "picture". In the story by Peter Bichsel, the lonely man ends up having so much trouble communicating with other people that he speaks only to himself. It is a very sad story.